Realistic error bounds for asymptotic expansions arising from integrals via resurgence

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Resurgence, emerging from the work of Dingle and Écalle, is now a fundamental tool in asymptotic analysis. Resurgence has been used successfully in the asymptotic theory of integrals, difference- and differential equations to obtain exponentially accurate approximations and to understand the Stokes phenomenon. Berry and Howls obtained convenient integral representations for the remainder terms of asymptotic expansions arising from an application of the method of steepest descents. These integral representations were then used by Boyd to obtain error bounds for such asymptotic expansions, in particular for the well-known Stirling asymptotic expansion of the gamma function. In this talk, I will give an alternative representation for the remainders involving the so-called terminant functions. These terminant functions were previously used for exponentially improved expansions and for the computation of the Stokes multipliers of differential equations, but, as I will show, they have a significant role in bounding error terms. It turns out that in many cases, the remainder can be bounded by the absolute value of the first omitted term of the expansion. I will provide applications for special functions such as the incomplete gamma function, the Airy functions and the Bessel functions (both large argument and/or large order). I will briefly compare the results with those arising from differential equation methods by Olver, and show that the new approach often gives much simpler and sharper bounds.